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UNSTEADY-STATE HEAT GENERATION AND HEAT TRANSFER IN SUPERCONDUCTING COMPOSITES

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Abstract—Approximate methods are developed to calculate the rate of heat generation with respect to time when current is suddenly discharged from part of a superconducting wire embedded in a copper sheath. The temperature field in the copper with respect to time is then found in the absence of any cooling from liquid helium surrounding the superconducting composite. Thus it is determined if the temperature at the end of the section of superconductor, from which the current has been excluded, rises above a critical temperature which will cause the region to propagate.

Unsteady-state film boiling of liquid helium is separately considered to determine if it can make a significant contribution to modifying the temperature field within the superconducting composite. The film boiling theory is put in a form readily applicable to liquids other than helium.

NOMENCLATURE

a, volume of copper per unit superconductor surface area;

B, constant defined by equation (34);

c, specific heat of helium vapour;

c_0, specific heat of helium vapour at y_0;

c_p, specific heat of copper;

F_C, F_1, F_R, F_S, functions of time defined by equations (22–25);

h_s, h_u, steady- and unsteady-state heat-transfer coefficients respectively;

H, magnetic flux;

I, current in superconductor;

I_p, current per unit perimeter of superconductor;

J, current density;

k, thermal conductivity of helium vapour;

k_0, thermal conductivity of helium vapour at y_0;

l, half length of superconductor from which current is excluded;

L, latent heat of vaporization;

m, mass of copper per unit superconductor surface area;

q_s, heat flux from surface adjacent to helium;

Q, heat;

Q_{LO}, heat due to current exclusion;

$\dot{Q}_C, \dot{Q}_1, \dot{Q}_R, \dot{Q}_S$, heat release rate into copper per unit surface area of superconductor due to current exclusion and during one-dimensional current diffusion, radial current diffusion and the steady state respectively;

r, radial coordinate;

r_0, radius of superconductor wire;

R, (r/r_0);

t, time;

t_{1R}, t_{1S}, t_{RS}, time at which form of solution changes. Subscripts indicate direction of change: 1 for one-dimensional current diffusion, R for radial current diffusion and S for steady state;

t_u, time period of unsteady-state film boiling;

T, temperature;
T_0, temperature of helium liquid–vapour interface;
T_s, temperature of solid surface adjacent to helium;
u, variable defined by equation (32);
v, velocity;
x, linear coordinate;
y, variable defined by equation (42);
y_0, value of y at helium liquid–vapour interface;
z, dimensionless time defined by equation (33);
z_{1R}, z at t_{1R}.

Greek symbols
α, thermal diffusivity of copper;
α_s, thermal diffusivity of the superconductor;
γ, helium vapour density;
γ_0, helium vapour density at y_0;
γ_L, helium liquid density;
η, $(T/T_0)^{\frac{3}{4}}$;
θ, temperature elevation;
ρ, electrical resistivity;
τ, time;
ϕ, defined by equation (43);
$\int_C, \int_1, \int_R, \int_S$, integrals defined by equations (29), (30), (31), (40).

1. INTRODUCTION

It is well known that the electrical characteristics of a single straight superconducting wire are more stable than one wound in a coil. The reasons need not concern this paper but the effect of instability is that magnetic flux is locally excluded from the wire, with a consequent generation of heat, which is sufficient to raise the temperature above a critical value at which the superconductor acquires an electrical resistance in the usual sense. The ohmic heating of the wire can then be sufficient to maintain the elevated temperature in competition with cooling by boiling liquid helium at the wire surface and even to cause the region of normal resistance to propagate, (see for example Whetstone and Roos [1]).

A practical method of maintaining the wire generally in a superconducting state to enable it to carry the very high currents of which it is theoretically capable is to embed the superconducting wire in copper and so provide low resistance shunt. This method is known to work but upon close examination the reasons are not so clear.

The diffusivity of current density normal to the direction of current flow is directly proportional to the electrical resistivity. Thus, since the resistivity of the superconductor above the critical temperature is large, the current is excluded from the superconductor in a negligibly small time—typically 10^{-8} s. The current will, however, diffuse rather slowly through the copper, taking more like 10^{-3} s, and the ohmic resistance losses, before the copper shunt can take full effect, will be large.

The magnitude of the current diffusivity in copper is 2·5 cm^2/s while that for thermal diffusivity is 5000 cm^2/s. Therefore any heat generated will spread rapidly over the copper cross-section which can be considered isothermal. The heat then has two paths to follow which are indicated in Fig. 1:

(1) It can pass from the copper into surrounding liquid helium or,
(2) Since the magnetic flux will have been excluded only from a certain length of superconductor, it can diffuse normal to the copper cross section to regions where the colder copper provides a heat sink.

It becomes clear that the initial virtue of the copper will be to obtain rapid diffusion of heat and to provide a heat sink rather than to act as a low resistance shunt. It would be hoped to design the superconducting composite to prevent the ends of the length of superconductor which had reverted to the normal condition from rising above the critical transition temperature. Then the normal region will not propagate.

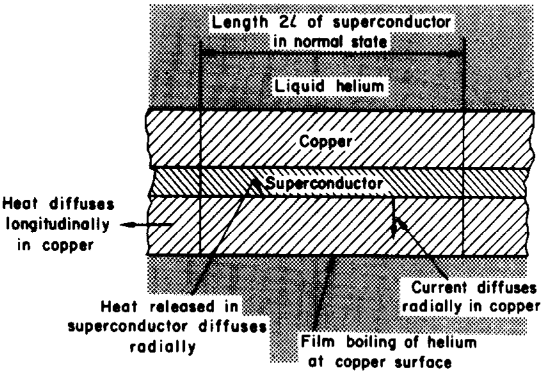

FIG. 1. Diagram describing the assumed model of events.

The nature of heat transfer to the liquid helium will be unsteady-state film boiling. Nucleate boiling can be discounted in most practical cases where the helium is under 1 atm pressure at 4·2°K and the surface temperature of the copper will be close to and most likely above the critical temperature of 5·2°K. It will be shown that the unsteady state will last about 10^{-3} s and so corresponds in magnitude with the current diffusion period within the copper.

This preamble has indicated that the various unsteady-state processes are complicated and interlocking. Nonetheless it will be shown that a reasonably accurate assessment of events can be made by simple desk calculation if heat transfer to the helium is ignored. This will assign temperatures to the system and then the importance of the boiling process can be judged using a film boiling analysis. Results of greater accuracy would require numerical work by a computer.

The model of events that is to be used for analytical purpose is implicit in the description given above and is illustrated in Fig. 1. The details of the model will appear in the assumptions to be stated in the later sections. First expressions will be derived giving rates of heat released in the copper. Then expressions will be given describing the diffusion of the heat through the copper. Finally an unsteady-state film boiling theory will be developed.

2. HEAT RELEASE RATE INTO COPPER

2.1. *Stored energy of magnetic flux*

The stored energy Q_{LO} per unit length of the cylindrical superconductor of radius r_0 is

$$Q_{LO} = \int_0^{r_0} \frac{H^2 r}{4} \mathrm{d}r \tag{1}$$

where H, the magnetic flux, is

$$H = 2\pi r J \tag{2}$$

where r is the radial co-ordinate, and J is the current density. Hence

$$Q_{LO} = \frac{I^2}{4}. \tag{3}$$

I is the current in electromagnetic units of say $g^{1/2}$ $cm^{1/2}/s$.

2.2. *Release of stored energy into copper*

The stored energy in the superconductor is usually a small part (about one fifth) of all the energy released in the unsteady-state current diffusion period. It also appears rather slowly in the copper due to the low thermal diffusivity of superconductor materials (about 5 cm^2/s compared with 5000 cm^2/s for copper) of practical superconductors. Therefore the following assumptions can be made without materially affecting the calculated copper temperatures, which are the final result.

It is assumed that the surface temperature of the superconductor is constant and that the temperature elevation θ above the surface temperature is of the form

$$\theta = f(R) \exp\left(-\frac{A\alpha_s \tau}{r_0^2}\right) \tag{4}$$

where α_s is the thermal diffusivity of the superconductor, τ is time, $R = (r/r_0)$ and A is a constant.

Substitution of equation (4) is the thermal diffusion equation

$$\frac{\partial^2 \theta}{\partial R^2} + \frac{1}{R}\frac{\partial \theta}{\partial R} = \frac{r_0^2}{\alpha_s}\frac{\partial \theta}{\partial \tau} \tag{5}$$

yields an ordinary differential equation in f and R with A as a parameter, which has a series solution for f in terms of R and A. The boundary conditions are then used to determine $A = 5{\cdot}8$. Thus it is found that $\dot{Q}_C$, the rate of heat release into the copper per unit surface area of superconductor is

$$\dot{Q}_C = \frac{2{\cdot}9\,\alpha_s}{\pi r_0^3} Q_{LO} \exp\left[-\frac{5{\cdot}8\,\alpha_s \tau}{r_0^2}\right]. \qquad (6)$$

2.3. *Heat release due to ohmic resistance in copper*

When the superconductor reverts to normal, it is assumed that the current enters the copper instantanously so that at first it is only present in a thin skin at the surface of the superconductor. The initial ohmic resistance of the copper under this assumption will thus be infinite and there must be some current sharing but this is ignored.

At first we can treat the current diffusion as one-dimensional, i.e. the problem is the same as for diffusion into a semi-infinite slab. At a later stage we can ignore the boundary formed by the surface of the superconductor and treat a problem in which all the current is assumed to originate at the axis of symmetry of an infinite solid. Further, for the purpose at hand, we can make a change from one solution to the other at the time where the rates of heat generation due to the two current distributions are equal.

A time will also be reached when the heat generation rate from one or other of the above solutions equals the rate when the current is uniformly distributed. This time can be taken as the end of the unsteady-state current diffusion period.

The equations of one-dimensional (x) and axisymmetrical diffusion of current density J are analogous to the thermal diffusion of current and are

$$\frac{\partial^2 J}{\partial x^2} = \left(\frac{4\pi}{\rho}\right)\frac{\partial J}{\partial \tau} \qquad (7)$$

$$\frac{\partial^2 J}{\partial r^2} + \frac{1}{r}\frac{\partial J}{\partial r} = \left(\frac{4\pi}{\rho}\right)\frac{\partial J}{\partial \tau} \qquad (8)$$

where ρ is the resistivity.

Equations (7) and (8) are solved, with the boundary conditions noted above and the condition that the total quantity of current is constant, to give

$$J = \frac{2I_p}{(\rho\tau)^{\frac{1}{2}}} \exp\left[-\frac{\pi x^2}{\rho\tau}\right] \qquad (9)$$

$$J = \frac{2\pi I_p r_0}{\rho\tau} \exp\left[-\frac{\pi r^2}{\rho\tau}\right] \qquad (10)$$

where I_p is the current per unit perimeter of the superconductor and is not to be confused with I in equation (3).

The energy dissipation per unit volume is ρJ^2 and thus the heat release rates $\dot{Q}_1$ for the one-dimensional solution and $\dot{Q}_R$ for the axisymmetric solution are given by

$$\dot{Q}_1 = \left(\frac{2\rho}{\tau}\right)^{\frac{1}{2}} I_p^2 \qquad (11)$$

$$\dot{Q}_R = \left(\frac{\pi r_0}{\tau}\right) I_p^2 \qquad (12)$$

also in the steady state we have

$$\dot{Q}_S = \frac{\rho}{a} I_p^2 \qquad (13)$$

where a is the copper volume per unit surface area of superconductor.

It is interesting to note that equation (12) is independent of the resistivity.

The changeover time from one solution to another is given by the following equations with the obvious subscript notation

$$t_{1R} = \frac{\pi^2 r_0^2}{2\rho} \qquad (14)$$

$$t_{1S} = \frac{2a^2}{\rho} \qquad (15)$$

$$t_{RS} = \frac{\pi r_0 a}{\rho}. \qquad (16)$$

As an example consider $\rho = 30$ cm^2/s, $r_0 = 0{\cdot}03$ cm and $a = 0{\cdot}15$ cm. Then $t_{RS} = 4{\cdot}5\ 10^{-4}$ s.

3. LONGITUDINAL DIFFUSION OF HEAT IN THE COPPER

In this section the diffusion of heat in the copper parallel to the axis of the superconductor is to be studied. Several important assumptions are made.

(a) The system is isothermal normal to the axis of the superconductor.
(b) The length of the normal superconductor is $2l$ and does not change.
(c) No heat is lost to the helium surrounding the copper.
(d) The thermal properties of copper are independent of temperature.

The last assumption is the most serious and is made so that the thermal diffusion equations become linear. Different temperature fields can thus be added to one another and desk computations are possible. The limitations imposed by the assumption are indicated by the variation of thermal diffusivity of a typical copper from 7000 to 3000 cm^2/s as the temperature changes from 4·2°K to 7°K—a practical range for consideration. The variation is not so severe that the magnitudes of temperatures cannot be calculated using the simplifying assumption and, of course, the effect of the variation of thermal properties on the magnitudes can still be argued qualitatively.

Consider the effect at time t of an increment of temperature $\mathrm{d}T$ added to the length $2l$ at the time τ. Carslaw and Jaeger [2] give the increment of temperature $\mathrm{d}\theta$ at a position x measured along the length of the wire from the centre of the length of normal superconductor.

$$\frac{\mathrm{d}\theta}{\mathrm{d}T} = \tfrac{1}{2}\left[\operatorname{erf}\frac{(l-x)}{2[\sqrt{\alpha(t-\tau)}]} + \operatorname{erf}\frac{(l+x)}{2[\sqrt{\alpha(t-\tau)}]}\right] \tag{17}$$

for $t \geqslant \tau$ where α is the thermal diffusivity of copper.

Attention will be concentrated on the temperature at the end of the length $2l$ where

$$\frac{\mathrm{d}\theta}{\mathrm{d}T} = \tfrac{1}{2}\operatorname{erf}\frac{l}{[\sqrt{\alpha(t-\tau)}]}. \tag{18}$$

Now equations (6), (11), (12) and (13) for the heat release rate are of the form

$$\mathrm{d}Q = F(\tau)\,\mathrm{d}\tau \tag{19}$$

and we have

$$\mathrm{d}T = \frac{\mathrm{d}Q}{c_p m} \tag{20}$$

where c_p is the specific heat of copper and m is the mass of copper per unit surface area of superconductor.

Thus equation (18) can be integrated

$$\theta = \frac{1}{2c_p m}\int_{t_1}^{t_2} F\operatorname{erf}\frac{l}{[\sqrt{\alpha(t-\tau)}]}\,\mathrm{d}\tau \tag{21}$$

where F for the various quantities of heat is given, using the usual subscripts, by the equations

$$F_C = \frac{2{\cdot}9\,\pi\alpha_s I_p^2}{r_0}\exp\left[-\frac{5{\cdot}8\,\alpha_s\tau}{r_0^2}\right] \tag{22}$$

$$F_1 = \left(\sqrt{\frac{2\rho}{\tau}}\right) I_p^2 \tag{23}$$

$$F_R = \frac{\pi I_p^2 r_0}{\tau} \tag{24}$$

$$F_S = \frac{\rho I_p^2}{a}. \tag{25}$$

Taking t_1, the initial time, to be zero we can write, ignoring θ_R for the time being

$$\theta_C = \frac{2{\cdot}9\,\pi l^2 I_p^2}{r_0 c_p m}\left(\frac{\alpha_s}{\alpha}\right)\int_C \tag{26}$$

$$\theta_1 = \left(\sqrt{\frac{2\rho}{\alpha}}\right)\frac{l I_p^2}{c_p m}\int_1 \tag{27}$$

$$\theta_S = \frac{l^2 I_p^2}{a c_p m}\left(\frac{\rho}{\alpha}\right)\int_S \tag{28}$$

where

$$\int_C = \int_0^z u \exp\left[-B(z^2 - u^2)\right] \operatorname{erf}\left(\frac{1}{u}\right) du \quad (29)$$

$$\int_1 = \int_0^z \frac{u}{\sqrt{(z^2 - u^2)}} \operatorname{erf}\left(\frac{1}{u}\right) du \quad (30)$$

$$\int_S = \int_0^z u \operatorname{erf}\left(\frac{1}{u} du\right) \quad (31)$$

where

$$u = \frac{[\sqrt{\alpha(t-\tau)}]}{l} \quad (32)$$

$$z = \frac{(\sqrt{\alpha t})}{l} \quad (33)$$

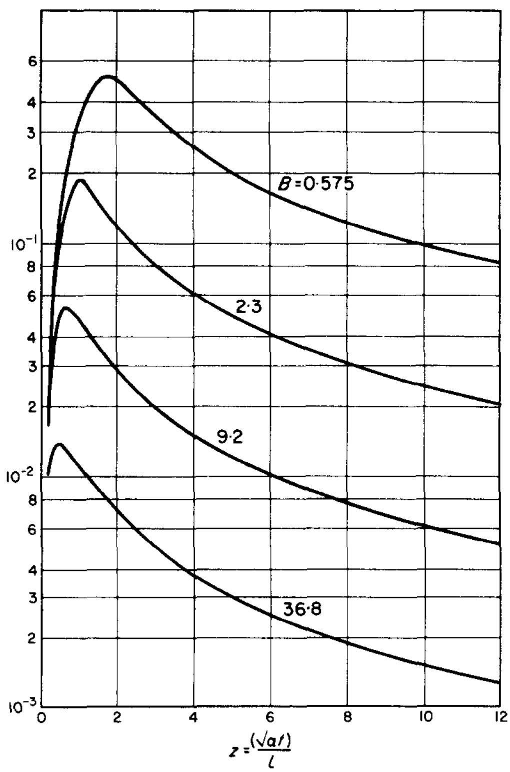

FIG. 2. Values of $\int_C$.

$$B = 5{\cdot}8\left(\frac{\alpha_s}{\alpha}\right)\left(\frac{l}{r_0}\right)^2. \quad (34)$$

Values of $\int_C$ vs. dimensionless time z are shown in Fig. 2 with B as a parameter. $\int_1$ is shown in Fig. 3 and it is seen that it tends to 1·75. This provides an upper limit to the temperature in the unsteady-state period.

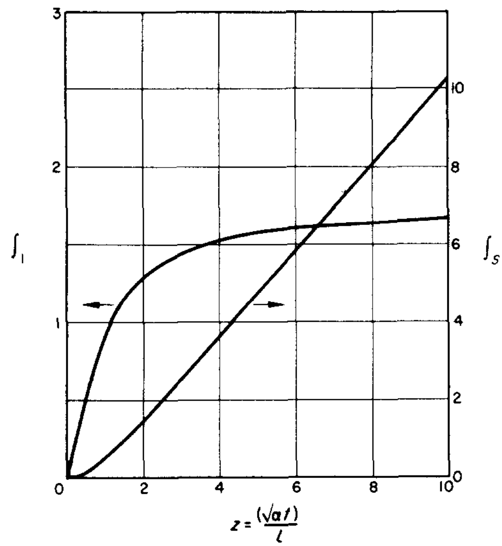

FIG. 3. Values of $\int_1$ and $\int_S$.

Figure 3 also shows the value of $\int_S$ and shows that it tends to become linear with respect to z or that θ_s tends to vary as the square root of time when the heat generation rate is constant in the steady state. Equation (31) has an analytical solution

$$\int_S = \operatorname{erf}\left[\frac{z}{2} + \frac{1}{z}\right] + \frac{z}{\sqrt{\pi}} \exp\left[-\frac{1}{z^2}\right] - 1 \quad (35)$$

from which we can find that

$$(\theta_s)_{t\to\infty} = \frac{2}{(\sqrt{\pi})} \frac{lI_p^2\rho}{ac_pm} \left(\sqrt{\frac{t}{\alpha}}\right). \quad (36)$$

We can also find solutions to equations (29) and (30) as z tends to zero and thus find the initial temperature rise

$$(\theta_C + \theta_1)_{t\to 0} = \frac{I_p^2}{c_p m}\left[1{\cdot}45\,\pi\frac{\alpha_s t}{r_0} + (\sqrt{2\rho t})\right]. \quad (37)$$

Equations (36) and (37) are valuable for comparing the initial rapid changes of temperature with the final slower changes.

3.1. *Temperatures during the radial diffusion of current*

During the period when the current diffusion is approximated by the axisymmetric system we must find the temperature

$$\theta_1]_0^t - \theta_1]_{t_{1R}}^t + \theta_R]_{t_{1R}}^t \quad (38)$$

where t_{1R} is the time at which a change is made from the assumption of plane diffusion to that of radial diffusion.

The first term of (38) has already been determined. The second term involves evaluating equation (30) between $u = 0$ and $u = (\sqrt{z^2 - z_{1R}^2})$. Now $(\sqrt{z^2 - z_{1R}^2})$ is usually small and thus erf $(1/u)$ does not change very much. Therefore equation (30) can be evaluated in steps assuming an average value of erf $(1/u)$ for each step. Usually only two steps and at the most three are needed for adequate accuracy. Equation (30), of course, has an analytical solution if erf $(1/u)$ is set constant.

A similar procedure is used for the third term of (38) where

$$\theta_R = \frac{\pi I_p^2 r_0}{c_p m}\int\limits_R \quad (39)$$

$$\int\limits_R = \int \frac{u}{z^2 - u^2}\,\mathrm{erf}\left(\frac{1}{u}\right)\mathrm{d}u. \quad (40)$$

3.2. *An example*

Figure 4 gives an example of the results of calculations using the methods given above. It shows the temperature in the copper at the end of the normal length of superconductor vs. the square root of time for the case where $a = 5r_0$. The original current density in the superconductor for the example is 80000 A/cm², $c_p = 2{\cdot}10^{-4}$ J/g °K, $\rho = 30$ cm²/s, $\alpha = 5000$ cm²/s γ (copper density) = 8·5 g/cm³. The temperature rise due to θ_C has not been added since it is small owing to the small value of $\alpha_s = 5$ cm²/s.

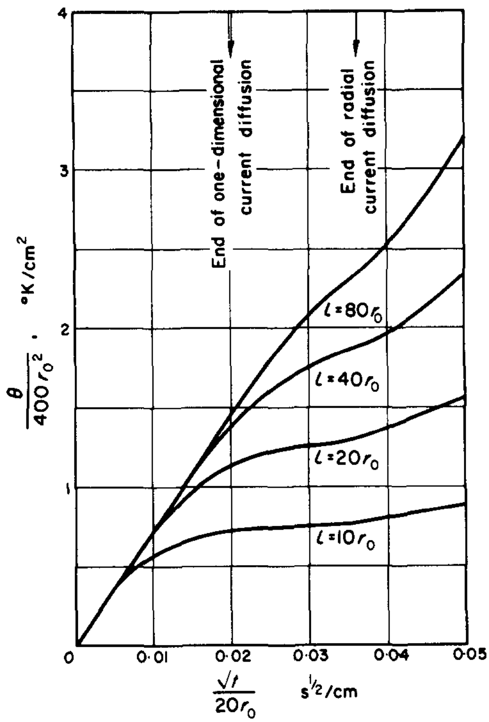

FIG. 4. Temperature elevation vs. time for $a = 5r_0$ and without heat from current expulsion of helium cooling.

Figure 4 shows that the temperature tends to reach a plateau but that the plateau gets eliminated as the length of the normal region increases. The shape of the curves in the steady-state region has been judged using equation (36) as a guide.

4. UNSTEADY-STATE FILM BOILING

Unsteady-state film boiling heat-transfer rates are to be calculated assuming that the solid surface has a constant temperature T_S and that the liquid is at its saturation temperature T_0.

Thus we have a vapour film with a variable temperature gradient and for which we can write the one-dimensional thermal conduction and convection equation

$$c\gamma \frac{\partial T}{\partial t} + c\gamma v \frac{\partial T}{\partial x} - \frac{\partial (k\, \partial T/\partial x)}{\partial x} = 0. \quad (41)$$

c, γ and k are the specific heat, density and thermal conductivity of the vapour and are all temperature dependent. v is vapour velocity and x is distance from the wall.

Hamill and Bankoff [3] obtained a solution to equation (41) assuming k was independent of temperature. Here we will use Hansen's [4] transformation which avoids this assumption.

$$y = \left[\frac{c_0}{4k_0\gamma_0 t}\right]^{\frac{1}{2}} \int\limits_0^x \gamma \, \mathrm{d}x \quad (42)$$

$$\phi = \int\limits_0^{(T/T_0)} \left(\frac{\gamma k}{\gamma_0 k_0}\right) \mathrm{d}\left(\frac{T}{T_0}\right) \quad (43)$$

where subscript 0 refers to properties at the temperature T_0. Thus equation (41) becomes

$$\left[\frac{c_0 k\gamma}{ck_0\gamma_0}\right] \frac{\mathrm{d}^2\phi}{\mathrm{d}y^2} + 2y\frac{\mathrm{d}\phi}{\mathrm{d}y} = 0. \quad (44)$$

Equation (44) can be numerically integrated from y_0, the position of the liquid–vapour interface, given a starting value for $(\mathrm{d}\phi/\mathrm{d}y)$. We use the condition that the rate of increase of the mass of vapour times the latent heat of vaporization L equals the heat conducted to the liquid–vapour interface and derive

$$\left(\frac{\mathrm{d}\phi}{\mathrm{d}y}\right)_0 = -\frac{2Ly_0}{c_0 T_0}. \quad (45)$$

Further we find the value of $(\mathrm{d}\phi/\mathrm{d}y)$ at the solid surface to be

$$\left(\frac{\mathrm{d}\phi}{\mathrm{d}y}\right)_s = -\frac{q_s}{T_0}\left[\frac{4t}{\gamma_0 c_0 k_0}\right]^{\frac{1}{2}} \quad (46)$$

where q_s is the heat flux from the solid surface. Thus the numerical integration will give $(\mathrm{d}\phi/\mathrm{d}y)_s$ and ϕ_s and hence equations (46) and (43) will give the heat flux from the surface and the temperature difference between the surface and the vapourizing liquid.

Equation (44) contains $(k\gamma/c)$ as an arbitrary function of ϕ. For many purposes and for the particular purpose at hand we can assume that c is independent of temperature and that

$$\left(\frac{\gamma}{\gamma_0}\right) = \left(\frac{T_0}{T}\right), \quad \left(\frac{k}{k_0}\right) = \left(\frac{T}{T_0}\right)^{\frac{3}{4}}. \quad (47)$$

Then

$$\phi = \tfrac{4}{3}[(T/T_0)^{\frac{3}{4}} - 1] = \tfrac{4}{3}[\eta - 1] \quad (48)$$

in which η is defined.

Equation (44) becomes

$$\frac{\mathrm{d}^2\eta}{\mathrm{d}y^2} + 2y\eta^{\frac{1}{3}}\frac{\mathrm{d}\eta}{\mathrm{d}y} = 0 \quad (49)$$

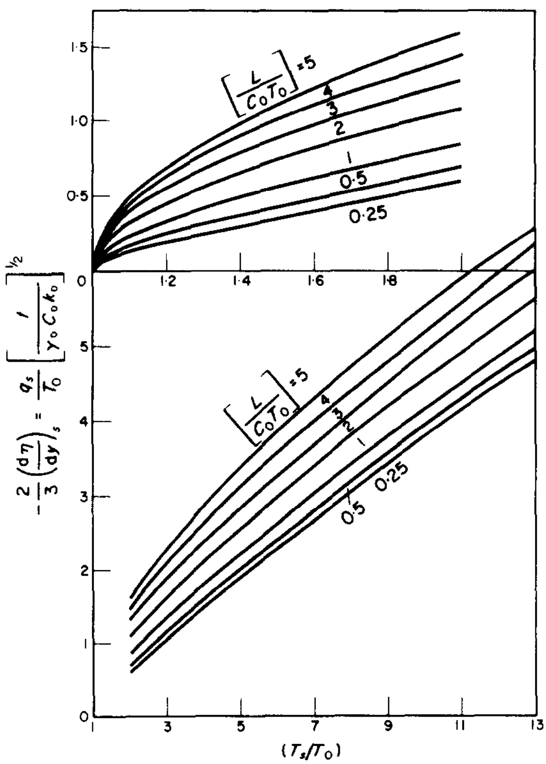

FIG. 5. Unsteady-state film boiling.

which has been integrated for the boundary conditions

$y = y_0, \quad \eta_0 = 1$ and

$$\left(\frac{d\eta}{dy}\right)_0 = -\frac{3}{2}\frac{Ly_0}{c_0 T_0}. \tag{50}$$

The results are shown in Fig. 5 in terms of the dimensionless heat flux

$$\frac{q_s}{T_0}\left(\frac{t}{\gamma_0 c_0 k_0}\right)^{\frac{1}{2}} \text{ vs. } \left(\frac{T_s}{T_0}\right) \text{ with } \left(\frac{L}{c_0 T_0}\right)$$

as a parameter.

4.1. *Application to helium at one atmosphere*

$(L/c_0 T_0)$ for helium at 1 atm is 0·965. The following table shows calculated unsteady-state heat-transfer coefficients.

Table 1. Heat-transfer coefficients for helium at 4·2°K

T_s	$h_u t^{\frac{1}{2}}$	h_s	t_u
4·282	1·175 10^{-2}	6·16 10^{-1}	3·63 10^{-4}
4·536	5·888 10^{-3}	2·48 10^{-1}	5·65 10^{-4}
4·99	3·935 10^{-3}	1·44 10^{-1}	7·45 10^{-4}
5·70	2·957 10^{-3}	9·82 10^{-2}	9·05 10^{-4}
6·73	2·380 10^{-3}	7·27 10^{-2}	1·07 10^{-3}
8·25	1·994 10^{-3}	5·85 10^{-2}	1·16 10^{-3}
10·49	1·711 10^{-3}	4·75 10^{-2}	1·30 10^{-3}
13·88	1·506 10^{-3}	4·05 10^{-2}	1·38 10^{-3}
19·03	1·339 10^{-3}	3·54 10^{-2}	1·44 10^{-3}
27·30	1·203 10^{-3}	3·17 10^{-2}	1·45 10^{-3}
65·86	9·857 10^{-4}	2·62 10^{-2}	1·42 10^{-3}
213·6	8·070 10^{-4}	2·25 10^{-2}	1·25 10^{-3}

T_s = surface temperature [°K]
h_u = unsteady state coefficient [W/cm² °K]
h_s = steady state coefficient [W/cm² °K]
t = time [s]
t_u = period of time for unsteady state [s]

The steady-state heat transfer-coefficient was calculated from the formula of Frederking, Wu and Clement [5] which fits available experimental information.

$$\left[\frac{h_s^3}{gkc^2\gamma(\gamma_L - \gamma)}\right]^{\frac{1}{3}} = 0{\cdot}3\left[\frac{L + c(T_s - T_0)}{c(T_s - T_0)}\right]^{\frac{1}{3}} \tag{51}$$

where vapour properties are those at the arithmetic mean film temperature, ρ_L is the liquid density and g is the gravitational constant.

The time period of the unsteady-state t_u is equal to the time at which $h_u = h_S$. It is seen to be about 10^{-3} s.

4.2. *Application to cooling of the superconductor composite*

It was stated in the introduction that the temperature field in the copper could not be calculated by simple means if boiling helium was included. This is partly because the copper temperature and thus the helium cooling varies with position along the composite as well as with time. No simple heat quantity, such as considered with respect to ohmic resistance heating, can be found which will give a temperature field to be subtracted from those already calculated.

Nonetheless the temperature elevations, such as shown in Fig. 4, can be used, in conjunction with the heat-transfer coefficients given in the table of the last section, to decide if the boiling heat-transfer process will remove a considerable or insignificant part of the heat generated in the superconductor composite. It is therefore possible to decide if the copper or the helium or both together should be considered as the immediate heat sink. In the long run of course the helium must remove all the heat.

In the composites that the author has studied it appears that, for values of $(2l)$ of about five times the diameter of the composite, both the copper and the helium are valuable as sinks. The answer will vary, however, with the design of the coil.

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Résumé—On expose des méthodes approchées pour calculer la vitesse de production de chaleur en fonction du temps lorsque le courant se décharge brusquement d'une partie d'un fil supraconducteur enveloppé dans une gaîne en cuivre. On obtient alors le champ de températures dans le cuivre en fonction du temps et en l'absence de refroidissement par de l'hélium liquid entourant l'ensemble supraconducteur. Ainsi, on détermine si la température à l'extrêmité de la section du supraconducteur, à partir duquel le courant a été expulsé augmente au-dessus d'une température critique qui provoquera la propagation de cette région.

L'ébulltion par film de l'hélium liquide en régime instationnaire est considérée d'une façcon séparée pour déterminer si cela peut contribuer sensiblement à la modification du champ de températures dans l'ensemble supraconducteur. La théorie de l'ébullition par film est mise sous une forme applicable directement à des liquides autres que l'hélium.

Zusammenfassung—Zur Berechnung der zeitabhängigen Wärmeerzeugung wurden Näherungsmethoden entwickelt für den Fall, dass einem Teil eines supraleitenden Drahtes plötzlich Strom entzogen wird. Der Draht ist von einer Kupferhülle umgeben. Das Temperaturfeld im Kupfer lässt sich ermitteln; es erfolgt dabei keine Kühlung des supraleitenden Körpers durch flüssiges Helium. Damit wird bestimmt, ob die Temperatur am Ende des Bereiches, dem der Strom entzogen wurde über eine kritische Temperatur ansteigt, was eine Ausdehnung des Bereiches zur Folge hätte.

In einer getrennten Betrachtung wurde geprüft, ob instationäres Filmsieden eine deutliche Änderung das Temperaturfeldes im supraleitenden Körper bewirken kann. Die Theorie des Filmsiedens ist in eine Form gebracht, die eine Anwendung auf andere Flüssigkeiten, ausser Helium, zulässt.

Аннотация—Разработаны приближенные методы расчета изменения со временем скорости тепловыделения при мгновенном отключении тока с участка проволоки из сверхпроводящего матариала, заключенного в медную оболочку. Находится изменение температурного поля со временем в медной оболочке при отсутствии охлаждения жидким гелием, в который погружен сверхпроводник. Таким образом определяется, поднимается ли температура на конце участка отключенного сверхпроводника выше критической, что вызывает увеличение этого участка.

Исследовалось также пленочное кипение жидкого гелия с целью выяснения степени его влияния на изменение температурного поля в сверхпроводнике. Теория пленочного кипения модифицирована для применения к другим жидкостям, кроме гелия.